

**NIST GCR 06-892**

**The Effects of Winds from Burning Structures On  
Ground-Fire Propagation at the Wildland-Urban  
Interface. Final Report**

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# The Effects of Winds from Burning Structures on Ground-Fire Propagation at the Wildland-Urban Interface

Ronald G. Rehm

## Abstract

The status of fire models for wildland fires is discussed in light of the corresponding models for fires in structures. The starting point for this discussion is a 1997 review paper by F. Albini entitled, “An Overview of Research on Wildland Fire.” Then, estimates of the parameters describing ground fires, structure fires and tree fires are presented and compared. Finally, an attempt is made to bridge the gap between these two types of modelling efforts by suggesting a simple, physics-based model for fire propagation at the wildland-urban interface (WUI). A grass fire is considered to be driven by both an ambient wind and entrainment winds generated by burning structures. Although modest, this model, which is based on the conservation of energy and mass, includes several interesting features and requires specification of several parameters. The dependence of the solution on the variation of some of the parameters is discussed.

## 1 Introduction

In 1997, Frank Albini [1] presented a talk entitled, “An Overview of Research on Wildland Fire,” at the Fifth International Symposium on Fire Safety Science, Melbourne, Australia. The attendees, members of the International Association for Fire Safety Science (IAFSS), were mostly engineers and scientists concerned with fires in structures, including myself. In his talk/paper, Dr. Albini discussed a much wider variety of issues than those discussed here. Quotes from this paper demonstrate the differences between the wildland-fire and the structure-fire communities. In this report, only issues related to mathematical model development are presented. Then, a simple model for wind-driven ground fires in a wildland-urban interface (WUI) setting, i.e., in the presence of burning structures, is presented, and results of the solution are discussed. It is hoped that this paper might stimulate better communication and cooperation between these two communities.

Unfortunately, on December 5, 2005, Dr. Albini passed away. His insight, technical capabilities and leadership will be missed! Albini is justifiably regarded as a leader in the wildland-fire community. Many of his papers were published in journals regularly read by members of the structure-fire community! Therefore, his 1997 paper was taken seriously by both communities. In the present paper, Albini’s original

assessment of the status of fire modeling is updated and an attempt is made to bridge the gap between the two communities by suggesting a model of wind-driven grass fires in the presence of burning structures.

Albini observed that while the interests of attendees “are focused mainly upon fire in man-made structures, many of its studies are relevant to, and applied in, modeling of wildland fire phenomenology. But the converse does not seem to be the case. Results of wildland fire research are seldom cited in the literature of fire safety research as it is done by this audience.” He further elaborated on this asymmetry in awareness of the research of one group by the other, noting, “the learning burden in this process will probably be greater for wildfire specialists than for traditional fire safety science researchers because the latter group strongly favors mathematical modeling of physical processes while wildland fire research traditionally incorporates a significant component of empiricism, often only weakly supported by conceptual models of underlying physical processes.”

While Albini discussed the importance and value of research and of understanding both wildland and WUI fires, he also cautioned that there are several pitfalls. For example, he stated that a major problem in the federal funding of wildland fire research arises because the subjects of fire effects and fire behavior compete for research funds. Since land managers, who are very influential in selection of the research projects and who are also expected to bring forth usable results in a timely fashion, research projects that rely heavily on empiricism are selected with fire effects greatly favored over those considering fire behavior. About wildland fire research, he further stated that, “I hope that in offering these snippets, I will offend as few of my colleagues as possible and entice as many new investigators as possible to this challenging, intriguing and poorly funded field of research.” Apparently, these funding realities have severely restricted the development of physics-based mathematical models of wildfire behavior and almost totally eliminated research on WUI fire.

Finally, Albini noted that, “Interest by the general public in matters of wildland fire safety has grown with increased exposure of affluent society to the hazards posed by building flammable structures in flammable wildland settings.” This quote is very important because it explicitly recognizes structures, if ignited, as part of the fuel system for the fire. The prevailing view regarding structures in land-management agencies seems to be that structures are surrounded by wildland fuel and isolated from other structures (see the schematic picture in Figure 1 reproduced from the recent GAO report cited below). When a wildfire encounters an individual structure, the structure either resists the thermal insult, or it ignites. Either way, according to this way of thinking, the structure is no longer of interest for determination of the wildfire behavior. In contrast, by this statement Albini recognizes explicitly that structures, when ignited, are part of the fuel system, and I infer from it that the fire behavior with the structure included, will be different than the fire behavior without the structure!

The objective of this paper is to trace briefly the history of physics-based mathematical modelling of fires in structures, and to suggest a simple example of a physics-based model for wildland urban interface (WUI) fires. In this way, I hope to illustrate that model development for structural fires can provide a useful approach to modelling wildfire and/or WUI fires without resorting to a complete Computational Fluid Dynamics (CFD) type description of these phenomena. Another way of stating this objective is to ask the question, What would a generalization of the Rothermel [3], [4] model look like that was: (a) physics based, (b) more complex and detailed, than the Rothermel model, but not as complex, mathematically and computationally, with intensive data requirements, as a CFD simulation?

In the next section, I briefly summarize some salient ideas in the development of early mathematical models of fire in a structure, the so-called zone models. In the following sections, I turn to considerations of modelling WUI fires. First, experimental observations concerning grass fires and tree fires are presented and compared with the corresponding information on compartment fires. These comparisons suggest a

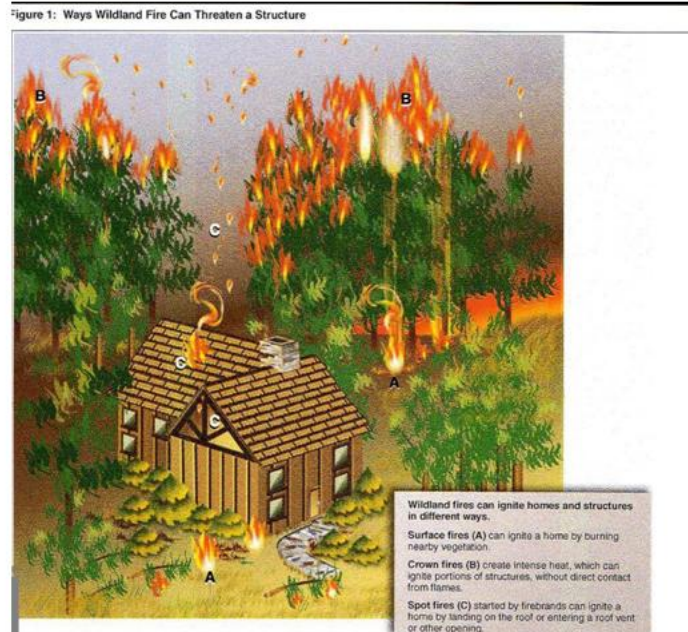


Figure 1: **Schematic diagram of a burning house in the woods as shown in the GAO report [2]. This perception of an isolated WUI house seems to be commonly held.**

possible direction for the development of a wildfire urban interface model based on wind-driven ground-fire spread. The lessons learned from development of the zone models suggests that a similar development process might be fruitful for the description of outdoor WUI fires. Hopefully, the wildland-fire community will agree.

## 2 Models for Structure Fires

Early studies on modeling fires include scientific and engineering leaders known for their expertise in heat transfer and fluid dynamics, such as Professor Hoyt Hottel, Sir Geoffrey I. Taylor, Dr. Phillip Thomas and Professor Howard Emmons. Each contributed technical insights and historical perspectives on fire research during its formative years in papers published in the Proceedings of the First International Symposium on Fire Research, sponsored by the National Academy of Sciences - the National Research Council, and held in Washington, 9-10 November 1959. The published volume of these proceedings was entitled, the "International Symposium on the Use of Models in Fire Research," which is remarkable because it appears to be the first and only symposium emphasizing the use of models in the conduct of fire research! The papers in this volume connect mathematical modelling and fire research in various application areas technically and historically. Contributors included scientists/engineers from the U.S. Bureau of Mines, the U.S. National Bureau of Standards and the U.S. Forest Service, as well as university and government researchers from England, France, Canada, and Japan, with interests in wildfire as well as structure fires.

In his description of the stimulation of fire research in the United States after 1940, Hottel [13] helps to explain the current fragmentation of fire-research activities remaining today. The penultimate statement in this paper suggests the difficulty of the subject; Hottel states: "A case can be made for fire being, next to life processes, the most complex of phenomena to understand."

Another paper in this volume by Thomas [6] reviewed the early developments in the modeling of fires in compartments. He discriminated between fire and combustion by noting that combustion systems have taps for the observer to control the continuous supply of fuel and air. Fire, on the other hand, has no such taps and allows for (or, in fact, requires) positive feedback between the essential components of fire: HEAT, FUEL, and OXYGEN. Thomas illustrated this distinction by invoking the so-called “fire triangle,” an early and often-used schematic diagram for fire. The diagram labels the corners of this triangle as the three critical components of fire, HEAT, OXYGEN and FUEL. Two of the sides are labelled to show the interactions, or feedback between these critical components: the side connecting HEAT and OXYGEN is described as “Buoyancy-induced flow,” while the side connecting HEAT and FUEL is described as “Thermal feedback to produce gaseous fuel.” Thomas reinforces the importance of these feedback mechanisms by citing a 1964 paper by the well-known fluid mechanician B. Morton, who remarks on the ability of a fire to entrain its own air supply and to produce fuel vapors needed to sustain burning from the solid or liquid as essential characteristics of fire.

Thomas also discussed models in general, noting there are two types, scaled physical models and mathematical models. Mathematical models of structural fires can be further divided into two categories, zone models and Computational Fluid Dynamics (CFD) models. He defined the word “model,” as a “representation, and, as in the visual arts, some representations are more complete or more idealized than others: neither an engineer’s nor an architect’s drawing is a complete representation but they allow certain purposes to be fulfilled and certain conclusions to be drawn.”

The definition I prefer originates with Marc Kac, the famous Cornell University and Rockefeller University applied mathematician, who described a mathematical model as a caricature. If well done, even a very simple caricature can be recognized immediately. Similarly, a mathematical model can be simple or complex, but captures the essence of the phenomenon! In Figure 2, two pictures of a house are drawn; each can be recognized immediately as a house. But the level of detail increases greatly from left to right, illustrating a second important feature of a mathematical model: more elaborate models are required as more detailed questions are asked!



**Figure 2: Mathematical models are caricatures that capture the essence of the phenomenon being modelled. They can be less elaborate (left) or more elaborate (right), depending upon the questions asked of the model and the resources devoted to it.**

The early modelling of fires in structures progressed by the study of fires in compartments. The importance of these compartment fires is that many of the processes occurring can be understood and approximated for engineering purposes by physics-based quasi-steady submodels, which can then be connected approximately through the laws of conservation of mass and energy in a self-consistent fashion. Because

each submodel occupies a region or zone of the compartment and approximates the physical processes differently, these models have become known as “zone models.” Important aspects of these submodels is that they could be related to satisfy the quasi-steady conservation laws, that they could be calibrated and validated using data from laboratory experiments, and that they were tractable based on the computational resources of the day.

Significant progress on the modelling of compartment fires began during the 1970s [7]. Relevant studies on these models were reported by Thomas [14], Quintiere [8]-[12], Friedman [16], Thomas [15], and Birk [17] and others. Two clear and useful books on fire research that describe zone models very well, are Quintiere [18] and Drysdale [19].

### **3 Models for WUI Fires**

Fire spread in a heterogeneous fuel environment, i.e. one having various types of fuels interspersed, is very complex and is not well understood. For example, at the so-called Wildland Urban Interface (WUI), both wildland fuels, such as trees, shrubs and grass or other ground fuels (pine needles, leaves, debris, etc.), and structural fuels are found intermixed. Unfortunately, while discussed at length and used regularly as justification for additional research, little effort has been devoted to serious examination of the physical basis for WUI fire spread.

A fuel system composed of wildland fuels alone may also be heterogeneous because it includes trees, shrubs and ground fuels intermixed. In general, wildland fuel has a three-dimensional (3D) structure, and this vertical distribution of fuel can be very important for determining fire behavior.

The fundamental 3D structure of the fuel is not addressed adequately in operational models of wildfires. Generally, the fuel load is specified as a mass (or energy) per unit area, so that it is effectively treated as painted on the ground. The ground surface may be regarded as having a three-dimensional topography, but the fuel variation with height above the ground is not considered. This simplification can be justified for large-scale wildland fires since the ratio of the height of the wildland fuels is small relative to the lateral extent of the fire (or relative to the height of topographical features). However, in general, this simplification is not justified, and the 3D nature of the fuel system should be accounted for.

The fuel system may be regarded as having two components, the ground-fuel portion and the vertical fuel structure. Propagation of fires in the ground fuel portion should be treatable as in past and as in current operational wildfire models, such as BEHAVE [20] and FARSITE [21]. For example, the propagation of fires in ground fuels in regions of significant topographical variation, such as grasses, leaves or pine needles, or slash, etc., with an occasional tree or shrub, or even an occasional structure, can probably be predicted reasonably well by this methodology. Furthermore, fires in large, nearly homogeneous tracts of trees in regions of significant topographical variation can also probably be modelled reasonably well by these same procedures. It is only when there are significant inhomogeneities in the fuel system, such as clusters or small stands of trees, or several structures, surrounded by grasses or other ground fuels, is it important to re-evaluate the limitations of the current methodology and to look for new mathematical models with which to predict fire behavior.

Over the past several years, models of fire behavior, based on the the equations for conservation of mass, momentum, energy and species and for radiative transport, have been developed and applied very successfully, particularly to fires in structures. These so-called field models are based on a partial differential and integro-differential equation field description of the conservation laws. There are many



examples of these field models, including FDS [22], [23], WFDS [24] and FIRETEC [25] and others. Colleagues in fire research at the National Institute of Standards and Technology (NIST) have been very active in the development of the Fire Dynamics Simulator (FDS) and WFDS, together with their visualization code, Smokeview [26]. Therefore, because of my familiarity with these models and codes, when I discuss field models, I have FDS and WFDS in mind.

For a fully developed room-fire, the heat released by the fire generates a buoyant plume, which in turn entrains ambient room air, heats it and pumps it up into a hot, smoky upper layer. The room then becomes stratified into the ambient-air lower layer and the heated upper layer, with the fire continuing to remove air from the lower layer through entrainment and adding the heated air plus combustion products to the upper layer as time proceeds. Zone models, which take advantage of physics-based mathematical submodels of these processes, are considerably simpler than the field models, usually resulting in a mathematical formulation consisting of nonlinear ordinary differential equations in time and complex algebraic relations between dependent variables. The model CFAST [27] is a most recent example from NIST of this class of models

Field models require considerably more data and computational resources and can provide substantially more detail about fire behavior than zone models: as noted above, zone models are more global and less detailed in their descriptions of fire growth and spread than field models. Therefore, for fires in buildings, field models, in principle, should be able to provide finer detail over smaller regions, for given computational resources, while zone models should be able to compute fire behavior while providing less detail over many more burning rooms.

In this spirit, assuming we need more detail than current operational models, but less than current field models, a simple model is proposed for wind-driven fire spread in heterogeneous, coupled fuel systems, namely ones with continuous fuel beds and also discrete fuel elements, on a scale of interest for WUI fires. While the model is modest and has significant limitations, it represents an attempt to carry out a physics-based coupling dynamically. The model can be viewed as analogous to zone models described above, and it is simple enough that it might be coupled into current operational models, such as BEHAVE [20] and FARSITE [21], in a usable fashion.

In many ways, this model can also be regarded as a dynamical extension of the model of Baum and McCaffrey [31]. In that paper, the authors derived a dynamically consistent solution to the mass and energy equations for an outdoor plume. This mathematically sophisticated solution is very useful because, as discussed below, it provides both scaling relations and detailed velocity profiles for this fundamental configuration. This model plume can be used as a primary component or zone for an equivalent outdoor zone model. Baum and McCaffrey [31] then applied their plume model in exactly that fashion to the study of mass fires. The plume model was also used by Ohlemiller and Corley [32] to estimate the thermally-induced winds generated during large-scale mass-fire experiments carried out by Forestry Canada. The estimated winds were found to be consistent with the measured winds during the experiments. Similarly, Trelles and Pagni [33] used this model to estimate the winds generated by multiple burning houses at various times during the Berkeley fire of 1991. These predicted winds were then compared with measured wind data at the same times, and it was found that significant wind changes occurred, consistent with the model predictions, at nearly the same times, when the number of burning houses increased dramatically. Specifically, over a 15 minute interval, from 11:45 AM to 12 noon on 20 October 1991 during the Oakland Hills fire, the number of houses burning was found to increase from 38 to 259.

The model proposed here has the advantage that, while very simple, it is physics based without the computationally and data intensive requirements associated with the field models. Furthermore, the model is dynamical rather than only static; i.e., it accounts for dynamical behavior, showing how a fire front

evolves with time.

There are two reasons for concentrating here on the coupling between a grass fire on the one hand and a burning cluster of structures on the other. First, recent experiments carried out in the Large Fire Laboratory at NIST have addressed the very fundamental issue of how the burning characteristics of an individual tree scale with tree height! In these experiments Douglas Firs ranging in height from about 1.3 m (4 ft) to about 6 m (18 ft) were burned, and the measurements have shown that the time profile of the heat release rate (HRR) in all cases is approximately triangular. Therefore, two parameters characterize these profiles, the peak HRR, denoted here as  $P$ , and its duration or half-width, denoted as  $\tau$ . The total energy (proportional to the weight loss) released during burning is  $\approx P\tau/2$ . The surprising conclusion of these studies is that the duration of the burns, to a first approximation (ignoring many important qualifications), remains of the order of one to two minutes, independent of tree size, while the peak HRR increases with the total energy released! If this relation is found to persist for larger trees and/or for other species, it has extremely important implications for modeling both wildland and WUI fires! In particular, it implies a poor coupling between structure fires and tree fires, since the time scale for a structure fire is measured in hours, whereas that for an individual tree is measured in minutes!

The second reason for examining this coupling is the recent detailed comparison, also carried out at NIST, between models of grass fires and experiments conducted in Australia. These studies have provided information used in the present study. In particular, they indicate that coupling between individual burning trees and the fire front for grass fires is probably small. If we consider a grassy landscape area of about 1000 m on a side, then the time scale for a windless grass fire line (rate of spread (ROS) 0.165 m/s) to propagate the 1000 m is about 1 hour 40 minutes. The rate of spread increases with ambient wind speed, and when the wind speed becomes of the order 60 miles per hour = 26.9 m/s, the ROS of the grass fire is about 15 m/s, yielding a fire front propagation time scale of  $\approx 1$  minute. Only at this rather high wind speed does the front propagation time become comparable to that of a tree burn under windless conditions. Now, the tree-burn time scale is likely to decrease substantially with wind speed. (However, there are not measurements to substantiate this assertion.) Therefore, it is most likely that there will be a mismatch between the grass-fire propagation time scale and the tree-burn time scale, and, therefore, the coupling between a burning individual tree and a grass fire is expected to be very small. Therefore, we will not consider grass-fire, tree-fire coupling at this stage of the modelling. (Note that I have said nothing about clusters or small stands of trees, which could burn in another way, since I am not aware of data on such burning clusters.)

In Section 4, data on heat content, burning rate and heat release rates (HRR) for various wildland and structural fuels is presented. An attempt is made to introduce rate processes in a way that will be useful for our proposed mathematical model. Then, a simple mathematical model is presented for the propagation of a grass fire driven by the wind field from an ambient wind plus one or more burning houses. The idea is to utilize the wind field determined from the Baum-McCaffrey plume to describe the wind field of a single burning structure. That model, which is summarized in the Section 5.1, requires the specification of a peak heat release rate (HRR) for the structure and provides both characteristic length and velocity scales, as well as the detailed normalized entrainment velocity as a function of distance from the center of the structure. The scaling of the effects on the fire front due to structure size, and, for multiple structures, structure spacing and number of houses, falls out of the analysis and is discussed in some detail in Section 5.2. In Section 5.3, the spread of an initial line fire is examined in the presence of this wind field, using a variant of Huygen's principle, to determine this front propagation. Features of this model are illustrated by examples in section 5.4. Finally, a summary and conclusions are presented in Section 6.

## 4 Heat Release Rates of Various Fuels

Rehm, et al [35] reviewed the literature on the potential energy content of various wildland fuels and compared these numbers with the potential energy content of structures. The purpose of that comparison was to estimate the density of structures required for the potential energy content to be equal to that of a particular wildland fuel. That work emphasized the importance of the potential energy content of the structures as part of the overall energy available, without regard for the dynamical processes required to ignite the fuels, sustain and propagate the fire, and with no estimate of the duration and completeness of the combustion processes.

This section extends that previous work by considering dynamic or time-dependent processes for heterogeneous fuels. Estimates of burning time scales, heat release rates (HRR) and propagation speeds for grass fires are compared with burning times, HRR and entrainment winds produced by ventilation-controlled structure fires. For an additional comparison, these same quantities are estimated for individual trees and crown fires.

### 4.1 Grass Fires

In a paper submitted to the International Journal of Wildland Fire, Mell et al [24] have compared data obtained from Australian grass-fire experiments with modelling results obtained by using the Wildand Fire Dynamics Simulator (WFDS) [24]. The experiments were carried out by CSIRO and were reported by Cheney et al [37] and Cheney and Gould [38]. The simulations were carried out with WFDS, a variant of the Fire Dynamics Simulator (FDS) code developed at NIST by McGrattan [22] and based on the techniques of computational fluid dynamics (CFD). This study is perhaps the most complete comparison between results from a modern mathematical/computational model and burn data for grasses.

Quantities from this grass-fire paper utilized in the present study are the rate of spread (ROS) of the fire line, the fire-line intensity and the total heat release rate (HRR). The relationship presented in the paper of Mell et al determines the ROS,  $r_w$ , in m/s for an Australian grass fire as a function of the ambient wind speed,  $V_a$  also in m/s:

$$r_w = 0.165(1 + 0.324 V_a) \quad [m/s] \quad (1)$$

The dimension of  $r_w$  is m/s and is shown in square brackets. (The notation of giving the dimension of a quantity in square brackets behind it will be regularly used in this report). Mell et al present the more general expression for the ROS  $r_w$ , which depends upon moisture content and fire-line length as well as the wind speed. However, for simplicity here, I utilize the limit of very small moisture and very long fire-lines. For that study, two example burns, designated as F19 and CO64, were examined in detail, and are listed below in tabular form. The values given in the table have a precision of only two significant figures, which reflects their uncertainty. Although other numbers may be transcribed below to the precision given in their sources, a precision of two significant figures is probably all that can be trusted in most cases.

Experiment	Fireline Intensity (MW/m)	Rate of Spread (ROS) (m/s)
<b>Case F19</b>	5.5	1.4
<b>Case CO64</b>	4.4	1.2

**Table 1: Data for testing WFDS model presented by Mell [24] from Australian grass-fire experiments carried out by CSIRO and reported by Cheney et al [37] and by Cheney and Gould [38].**

For comparison, Table 2 is reproduced from Rehm et al [35]. This table consists of data originally presented by Albini [36] with some of the data converted to SI units that are more convenient for the comparison. For example, the spread rate data in column 2 was given originally by Albini in miles per hour; it is converted to meters per second in column 3. The ROS and the fire-line intensity allow one to determine the total HRR and a burn duration for a specified length of fire line.

Types of Wildfire	Spread Rate (mi/h)	ROS (m/s)	Intensity (MW/m)	Fuel Energy Density (GJ/hectare)
<b>Ground Fire</b>	0.00003	.0000083	0.00001	12
<b>Surface Fires</b>				
Marginal Conditions	0.01	0.003	1	3.7
“Good” Conditions	10	2.77	10	36
Grass Fires	20	5.54	1	1.8
Debris Fires	1	0.277	10	370
<b>Crown Fires</b>	3	0.833	10	121

**Table 2: Types of Wildfires, Rate of Spread (ROS) and Intensities as reported by Albini [1]. Also shown are fuel energy density implied by these values.**

## 4.2 Structure Fires

An estimate of the energy release rate during a house fire in the 1991 Oakland Hills fire was made by Trelles [39] and by Trelles and Pagni [33]. According to these estimates, a house burns at a peak rate of 45 MW for 1 h (yielding about 160 GJ), and then dies down over another 6 h period. The die-down of the fire is approximated as two steps, one 10 MW for 3 h and the last as 5 MW for 3 more h. The total burn time is 7 h, and the total energy released by the house is 324 GJ. If, as assumed also, there is brush around each house which releases another 5 MW for one h, then an additional 18 GJ of energy will be released. If the house is assumed to be 15 m by 15 m by 5 m, then we estimate the total potential fuel loading per unit area to be of order 1.4 GJ/m<sup>2</sup>, the peak HRR per unit area to be of order 0.2 MW/m<sup>2</sup>. For comparison, oil yields a heat release rate per unit area of approximately 2 MW/m<sup>2</sup>, see McGrattan et al. [23] and Baum et al. [41].

Confirmation of this estimate for the magnitude of the peak HRR for a burning structure can be found in the chapter on compartment fires in the book on fire behavior by Quintiere [9]. Here, Quintiere describes the stages of a fire in a compartment and estimates the peak heating rates possible during the latter stages of a compartment fire, when the fire is fully developed and ventilation limited. Ventilation limited merely means that the burning rate is restricted by the amount of air entering the enclosure, and this amount is determined by the size of the vents in the enclosure. During this period, the flow in and around the enclosure is driven by buoyancy, which is generated by the burning taking place both inside and outside the compartment.

In one example, Quintiere estimates a total HRR of 9 MW for a compartment in which the fuel load is taken to be proportional to the floor area, that in this case is 12 m<sup>2</sup>. He points out that, this peak HRR could increase to over 60 MW if the fuel was proportional not only to the floor area, but to the whole inside area of the compartment. Such a lined compartment might be much more characteristic of a cabin in the woods, for example, for which the wall and ceiling were constructed of wood. Furthermore, a multi-room structure, with a fuel loading of the more modest type, producing 9 MW for each room, could also easily exceed the

roughly 50 MW peak HRR estimated by Trelles and Pagni [33]. Therefore, based on compartment fire analysis, it seems very plausible that structure fires could have peak HRR reaching several times this estimate of 50 MW, and the duration of these peak HRR would be measured in tens of minutes or hours.

These estimates say nothing about the fact that the roof of the structure might develop a hole or even collapse under prolonged vigorous burning. In that case, the fire might then resemble more a burning crib than an enclosure fire. They also say nothing about the effects of winds on peak heat release rate or burn duration. It seems likely that winds would increase the peak HRR and reduce the burn duration, but the magnitude of these changes is not known. For our purposes here, the estimates above will be used without trying to assess these other effects.

### 4.3 Tree Fires

It is well known that trees will only undergo ignition and sustained combustion if they are dry enough, and then only the needles and the small roundwood on conifers or leaves and small roundwood on deciduous trees will burn in general. Furthermore, the mass loss during burning is known to be proportional to the total energy released during burning, and this energy release rate is approximately 17-20 MJ/kg times the mass loss rate. However, the time dependence of the heat release rate (HRR) for individual trees of different sizes is not well established.

A series of experiments in the Large Fire Laboratory at NIST has begun to address some critical questions of the burning characteristics of individual trees, specifically Douglas Firs. First experiments carried out on very dry trees have determined that the profile of the HRR versus time for Douglas Firs ranging in height from 1.3 m to 6.6 m (4 ft to 20 ft) is approximately triangular, and can be characterized by two quantities, the peak HRR, designated here as  $R_0$  and the burn duration  $\tau$ . Furthermore, since the burn duration in all cases is nearly the same, 1 to 2 minutes, independent of height, the peak HRR increases proportionally to the mass (energy) loss during the burn.

ICFME Plot	Size (m)	Wind Speed (m/s)	Total fuel consumption (kg/m <sup>2</sup> )	ROS (m/s)	Fire line intensity (MW/m)
<b>A</b>	75 × 75	4.4	4.605	0.937	77.688
<b>1</b>	150 × 150	2.8	4.601	0.588	48.697
<b>2</b>	150 × 150	2.2	2.831	0.263	13.402
<b>3</b>	150 × 150	3.1	5.062	0.405	36.902
<b>4</b>	150 × 150	4.1	3.975	0.743	53.162
<b>5</b>	150 × 150	3.5	5.548	0.482	48.134
<b>6</b>	150 × 150	4.8	4.326	0.600	46.721
<b>7</b>	150 × 150	4.8	4.504	1.153	93.476
<b>8a</b>	150 × 150	3.1	4.708	0.405	34.321
<b>8c</b>	150 × 150	4.0	—	0.900	76.270
<b>9</b>	150 × 150	6.9	4.284	1.163	89.681

**Table 3: Data from the International Crown Fire Modelling Experiment (ICFME) carried out between 1997-2000 in the Canadian Northwest Territories as reported by Stocks et al. [43].**

A series of crown-fire experiments was carried out over the period 1997-2000 in the Canadian Northwest Territories as part of the International Crown Fire Modelling Experiment (ICFME), and the results of these

experiments were reported in a special issue of the Canadian Journal of Forest Research. An overview of these experiments, together with a large amount of detailed data, was reported in this issue. Table 3 contains some of this data as reported by Stocks et al. [43] to four significant figures. As noted above, the precision of this data is most likely two significant figures only.

Other data from these experiments was reported by Butler [42], and a summary of this data is given in Table 4.

Exp.	Height (m)	Wind (m/s)	ROS (m/s)	Total burnable mass/area (kg/m <sup>2</sup> )	Overstory stems/area (1/m <sup>2</sup> )	Mass/stem (kg)	Energy/stem (MJ)	HRR/stem (MW)
<b>Plt4</b>	11.1	4.17	0.74	0.98	0.41	2.4	41	0.41
<b>Plt9</b>	10.7	6.94	1.16	0.80	0.41	1.95	33	0.33

**Table 4: Data reported by Butler [42] from 1999 Canadian Northwest Territories crown-fire experiments together with some inferred quantities.**

#### 4.4 An Interpretation of Crown-fire Propagation

A very simplistic conceptual model for crown-fire propagation permits interpretation of the data obtained from 1999 Canadian Northwest Territories crown-fire experiments reported by Butler [42]. An estimate of various properties of these crown fires can be made, and the consistency of these estimates can be examined. For example, for a given line-fire intensity over a specified length of the crown fire front, the total heat release rate can be estimated. To make this calculation, an estimate of the depth of the crown fire line using the burn properties (peak HRR and burn duration) of an “average” tree for the trees of the type and height in these Canadian tree stands must be made. As in the NIST tree-fire experiments, we take a burn duration of 1 to 2 minutes and a peak HRR proportional to the total energy released during the burn for an individual tree.

Consider a one-dimensional crown line fire propagating uniformly along the x-axis. For this very simplistic model, we make several rough assumptions: take all trees to be the same, uniformly spaced with very dry burnable mass. Make the following definitions:

$h$ = the uniform space between trees (in both the  $x$  and  $y$  directions),

$r$ =the ROS of the fire,

$q$ = the peak HRR for a single tree,

$m$ =the burnable mass of each tree,

$\tau$ =the burn duration for each tree,

$E$ =the energy per unit tree mass released during burning, 17-20 MJ/kg.

The total energy released in each tree burn is  $q\tau/2 \approx mE$ . During the burn time for one tree, the fire front progresses a distance  $d = r\tau$ , igniting a number  $N$  trees,  $N = d/h = r\tau/h$ . Therefore, the fire line intensity is  $I = qN/h = (r/h)(mE/h)$ .

The mean value of the ROS in the Canadian crown-fire experiments, as reported by Stocks et al [43] in Table 2, is 0.69 m/s, the mean value of the fire-line intensity is 56 MW/m and the mean mass consumed per unit area is 4.4 kg/m<sup>2</sup>. If we take as the energy per unit mass of the wildland fuel in these experiments to be 20 MJ/kg, then, we can determine consistency of the equation relating the fire-line intensity to the consumed mass and the rate of spread of the line fire:  $I = mrE$ . Plugging in the values of the mean values

of these quantities, we find that  $4.4 \text{ kg/m}^2 \times 0.69 \text{ m/s} \times 20 \text{ MJ/kg} \approx 62 \text{ MW/m}$ , and this value is within about ten percent of the mean fire-line intensity given in Table 2! Hence, these estimates, while crude, seem plausible.

The ROS reported by Butler in the Canadian crown-fire experiments, Table 3, was 0.74 m/s for Plot 4 and 1.2 m/s for Plot 9. Butler also states that the flaming combustion lasted between 12 s and 35 s. Therefore, we expect the depth of the fire front to vary between  $d = 0.74 \times 12 = 8.9 \text{ m}$  and  $0.74 \times 35 = 26 \text{ m}$  for Plot 4 and  $1.2 \times 12 = 14 \text{ m}$  and  $1.2 \times 35 = 41 \text{ m}$  for Plot 9. Take as a representative value for the fire line depth for both experiments the average value of all of these, namely,  $d = 22 \text{ m}$ . Using 4100 tree stems per hectare reported in the abstract, a distance of  $h = 1.6 \text{ m}$  is obtained, whereas,  $h = 1.2 \text{ m}$  for the value of 0.65 tree stems per  $\text{m}^2$  for Plot 4 given in Table 2, and  $h = 1.3 \text{ m}$  for 0.56 stems per  $\text{m}^2$  for Plot 9 reported in Table 2. Similarly, take the tree spacing for both experiments to be the average of all three of these values, namely,  $h = 1.4 \text{ m}$ . Furthermore, take the ROS be the average of both experiments reported in Table 2,  $r = 1.0 \text{ m/s}$ , the burnable mass per tree to be the average  $m = 2.2 \text{ kg}$ . Finally, take the burn duration for an individual tree to be about  $100 \text{ s} = \tau$ , as found in the NIST experiments. Then,  $q = mE/\tau = 0.37 \text{ MW}$ ,  $N = d/h \approx 16$ , and  $I = qN/h \approx 4.3 \text{ MW/m}$ . Butler also reported that the experimental plots were either  $75 \text{ m} \times 75 \text{ m}$  or  $150 \text{ m} \times 150 \text{ m}$ . Then the total HRR was either  $4.3 \text{ MW/m} \times 75 \text{ m} = 320 \text{ MW}$  or  $640 \text{ MW}$ , and the total burn durations ranged between  $75 \text{ m}/(1.16 \text{ m/s}) = 65 \text{ s}$  and  $150 \text{ m}/(0.75 \text{ m/s}) = 200 \text{ s}$ . All of these estimates, while crude, seem consistent or plausible.

## 5 A Simple Model for WUI Ground-Fire Spread

Models that address WUI fire spread must necessarily be complex because of the heterogeneity of the fuel. The usual conceptual models for the interaction of a wildfire with structures regard the structures as isolated in the middle of wildland fuels. See Figure 1 for example, an illustration taken from the Government Accountability Report [2]. This figure implies a density of houses that is so low that the burning of these houses has no effect of the progression of the wildfire. In fact, the usual attitude toward structures is that they either remain untouched by the wildfire moving past them, or they are ignited by the wildfire, but only begin burning vigorously well after the passage of the fire front. In either case, the burning of the house has no influence on the progression of the fire front!

In reality, often there are many structures in addition to the wildfire itself that can contribute to the overall fire spread. For example, in Figure 3, a photograph of the Cedar Fire about to invade the Scripps Ranch residential community is shown. If the structures in this figure were to be ignited by brands (or other means), then the fire front would be invading a community with burning structures.

Here a model for the wind-driven spread of a ground fire in a WUI setting is presented. While very simple, it does offer some insight into one important growth mechanism, ground-fire spread by a combination of ambient wind and the wind generated by burning houses. This model provides quantitative estimates of the effects of wind-driven ground-fire spread in the presence of burning houses. Nothing is said about how the houses or the fire front have been ignited.

We will assume an array of uniformly spaced, identical houses for  $y \geq 0$ , and a constant ambient wind  $V_a$  in the positive  $y$ -direction blowing a grass fire front across the houses from below; see the schematic diagram in Figure 5.

In the remainder of this section, a mathematical formulation of this conceptual model is presented.



Figure 3: **Photograph of the Cedar Fire about to invade the Scripps Ranch residential community.**

## **5.1 Plume Model of Baum and McCaffrey**

The paper by Baum and McCaffrey [31] is the starting point for the analysis reported here. In that paper there is a fundamental analysis of the structure of a plume and its associated flow field produced by a pool fire in a quiescent atmosphere. An empirical correlation for centerline temperature and velocity was determined from the compilation of data obtained from a large number of pool-fire experiments carried out by many investigators over a wide variety of pool-fire diameters. Based on the buoyant, inviscid equations of motion and this correlation, the analysis obtains the scaling relations for the characteristic length and velocity scales for a pool-fire plume. Furthermore, a detailed velocity profile is determined from a solution to these equations.

Our analysis utilizes this plume model to describe the flow field generated by a single burning house and to estimate the effects of this flow field on the progression of a ground fire. The import of the analysis, I believe, is that it demonstrates with a simple physics-based model and an inexpensive computational scheme that a house, once ignited, becomes part of the fuel system and affects fire-line progression. It also allows us to investigate the changes in the fire-line spread as various parameters are changed, such as the



number and location of burning structures.

The model of Baum and McCaffrey [31] is for a single buoyancy-driven plume in an inviscid, quiescent fluid of density  $\rho_0$ , temperature  $T_0$  and pressure  $P_0$  at ground level. The magnitude of the heat release rate of the source is designated as  $Q_0$ , and the specific heat of the air is denoted as  $C_p$ . The model starts with the equations for mass, momentum and energy, assuming axial symmetry. The velocity field is then decomposed by a fundamental theorem of vector analysis into two components, one specifying the divergence and the other the curl. The divergence of the velocity results from thermal expansion of the gas, and the curl is the vorticity, and these components can be related to the plume centerline temperature and velocity correlations. From this analysis, the following set of scaling relations arise:

$$\begin{aligned} D^* &= \left( \frac{Q_0}{\rho_0 C_p T_0 \sqrt{g}} \right)^{2/5} \\ V^* &= \sqrt{g D^*} \end{aligned} \quad (2)$$

where  $D^*$ =length scale [m],  $Q_0$ =heat source [W],  $\rho_0$ =ambient density [ $\text{kg/m}^3$ ],  $C_p$ =specific heat at constant pressure [J/kg],  $T_0$ =ambient temperature [degrees K],  $g$ =acceleration of gravity [ $\text{m/s}^2$ ] and  $V^*$ =velocity scale [m/s].

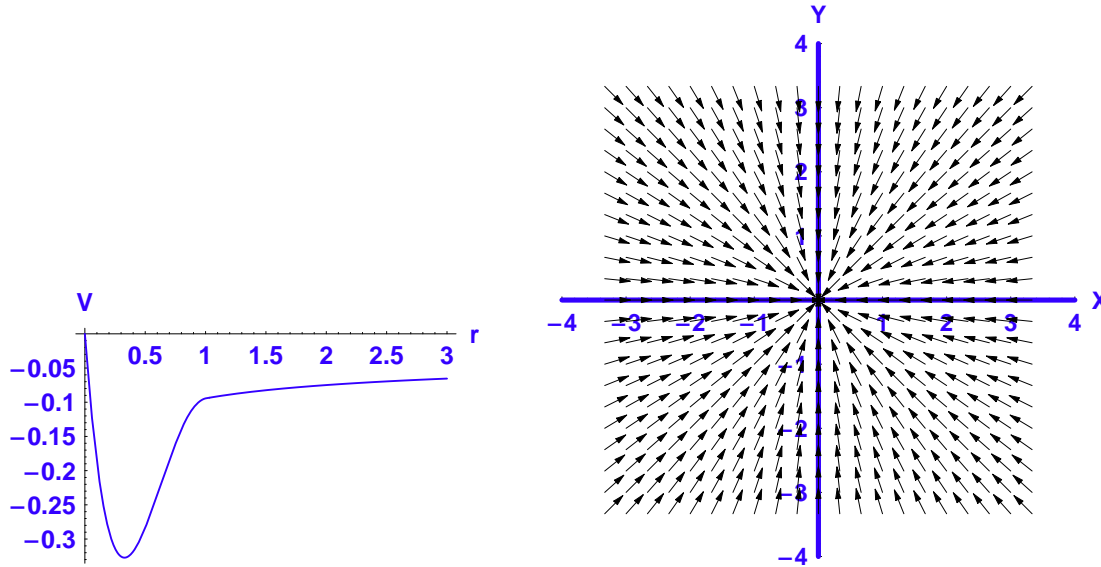


Figure 4: **LEFT: Normalized entrainment velocity induced by a structure fire at ground level versus normalized radial distance from the center of the fire. RIGHT: Entrainment velocity vectors at ground level induced by a burning structure.**

Finally, a detailed solution for the velocity field, which is valid both inside and outside the plume, was found by Baum and McCaffrey. This velocity field at ground level is shown in Figure 4.

## 5.2 Scaling Relations

The scaling relations obtained from the plume analysis of Baum and McCaffrey can be used to estimate the importance of burning structures on wind-blown fire spread in the WUI. Below, tables of these

characteristic quantities are presented, followed by the definitions and units in square brackets, are given for structures first, then for wildland fuel.

### Structure Fire

Structure Fire	Fire Size	Fire Size	Fire Size	Fire Size
$Q_0 \cdot 10^{-6}$ [MW]	25	50	100	500
$D^*$ [m]	7.5	12	19	55
$V^*$ [m/s]	8.6	11	14	23
$\tau_s$ [s]	$4 \times 3600$	14400	14400	14400
$E_s$ [GJ]	360	720	1440	7200

**Table 5: Characteristic scaling quantities for cases of structure fires.**

$Q_0$  = heat release per unit time for the structure [W],

$D^*$  = length scale of the plume (characterized by  $Q_0$ ) [m],

$V^*$  = velocity scale of the plume (characterized by  $Q_0$ ) [m/s],

$\tau_s$  = burning time for the house [s],

$E_s$  = total burnable energy in the structure [GJ].

### Wildland Fire

Wildland Fire	Grass	Grass	Grass	Trees
Structure density [No./acre]	4	4	1	0.25
$L$ [m]	32	64	128	32
$I$ [MW/m]	5	5	5	10
$U$ [m/s]	1	1	1	1
$\tau$ [s]	36	36	36	36
$m_f$ [kg/m <sup>2</sup> ]	0.06	0.06	0.06	0.6
$h$ [MJ/kg]	20	20	20	20
$e$ [MJ/m <sup>2</sup> ]	1.2	1.2	1.2	12

**Table 6: Characteristic scaling quantities for cases of wildland fires.**

$L$  = length between structures (the spatial period) [m],

$I$  = the fire line intensity [MW/m],

$U$  = the rate of spread (ROS) of the fire front [m/s],

$\tau$  = the passage time for the fire front [s],

$m_f$  = the mass of wildland fuel per unit area [kg/m<sup>2</sup>],

$h$  = burnable energy per unit mass of wildland fuel [MJ/kg].

(Note: 1 acre=4047 m<sup>2</sup>, and 1 hectare=10,000 m<sup>2</sup>.)

These characteristic quantities, coupled with the model described above, lead to several interesting conclusions. We consider two cases, that of a fire front propagating past a single burning structure and that of a fire front propagating past an array of burning structures. This latter case is shown schematically in the figure below, where the front is initially a straight line along the x-axis, which can be blown by an ambient wind toward the array. The array is considered to be rectangular with M structures in the x direction and N structures in the y direction, with the single house being the special case of M=N=0.

First, consider the ratio of the entrainment velocity  $V^*$  to the rate of spread  $U$  of the fire front in the

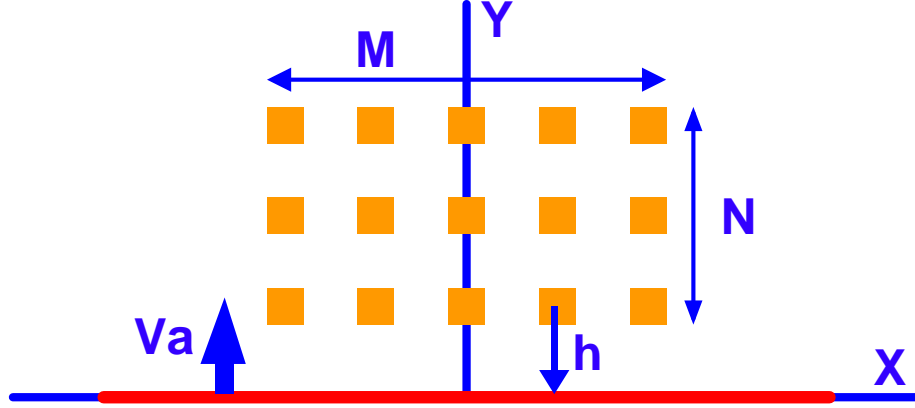


Figure 5: **A schematic of an array of uniformly spaced, identical houses with a constant ambient wind  $V_a$  blowing a fire front across the houses.**

absence of wind as a function of the heat release rate (HRR) for an single burning structure. This ratio is plotted at the left of Figure 6. Note, that this ratio is about 10 for even the rather modest value for the HRR of 20 MW, and increases with an HRR to about 35 at the large value of 2000 MW for the HRR. This relatively large ratio can be considered to result from the fact that the fuel in the structure is concentrated, or discrete, whereas that in the wildland fuel is uniformly distributed over the area.

Next, for an array of regularly spaced burning structures, separated by a distance  $2L$  in each direction, the y-component of the total entrainment velocity at a  $x, y$  is

$$v_{total}(x, y, L, V^*, D^*) = \sum_{j=0}^N \sum_{i=-I}^I V^* v \left( \frac{\sqrt{(x-2iL)^2 + (y-2jL)^2}}{D^*} \right) \frac{(y-2jL)}{\sqrt{(x-2iL)^2 + (y-2jL)^2}} \quad (3)$$

where, we have used  $I \equiv (M-1)/2$  in all cases presented. Equation (4) is an expression for the y-component of the characteristic entrainment velocity as a function of several parameters. Plots of this expression are shown at the right in Figure 6. The plot shows the enhancement of the entrainment velocity over the fire front ROS for multiple structures as a function of the distance  $r = y$  from the center of the middle structure in the first row of these structures and the other parameters, their separation  $2L$ , the HRR  $Q_0$  of each, and the total number  $M$  in the x-direction by  $N$  in the y-direction. This plot shows the collective effect of the the array of burning houses. The base case in this set of plots is for  $L = 60$  m,  $M = N = 5$  houses and  $Q_0 = 50$  MW; it is the curve labelled as  $L = 60$ . Naturally, as the number of houses,  $M$  by  $N$ , increases, the enhancement increases. Also, as the HRR,  $Q_0$ , of each house increases, the enhancement increases. As the spacing,  $2L$ , between houses increases, the entrainment decreases. The spacing  $2L$  can also be interpreted as the housing density. As the housing density increases, the spacing decreases; this point is discussed in Section 5.4.

The cluster of three curves labelled by  $L$  show what happens when the base case  $L$  is doubled or halved, whereas the other two curves show what happens when  $M = N$  is doubled or when  $Q$  is doubled. A somewhat surprising feature of these plots is the magnitude of the enhancement. The magnitude of the collective effect is a result of the fact that the entrainment velocity for each plume drops off with distance from the the plume center as one over the distance to the one-third power. However, all of these effects (and the relative surprises) can be inferred from the analysis of the plume model given by Baum and McCaffrey [31].

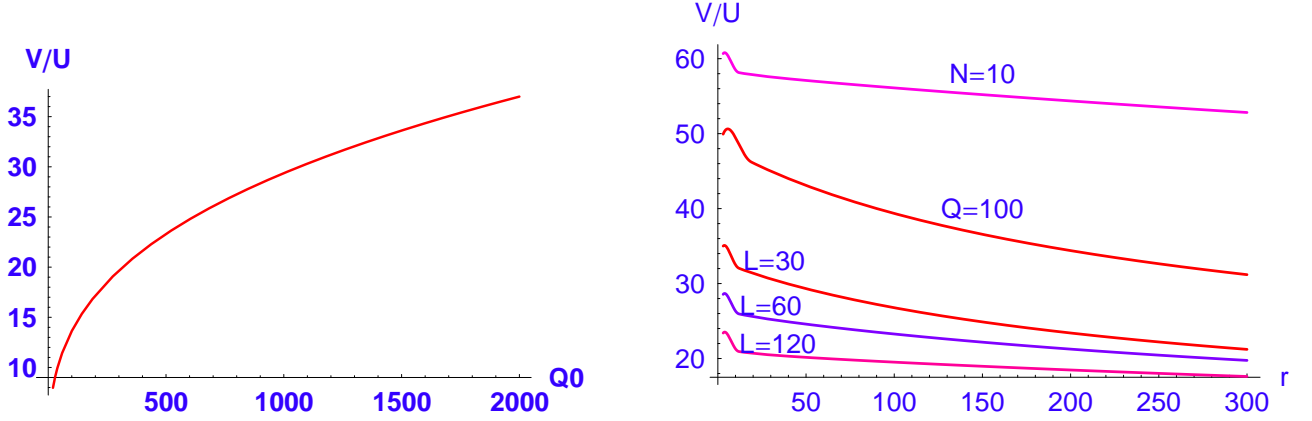


Figure 6: **LEFT:** The ratio of the characteristic entrainment velocity to the rate of spread (ROS) of the fire front as a function of the heat release rate (HRR) of a burning structure in megawatts (MW). **RIGHT:** The ratio of the characteristic entrainment velocity to the rate of spread (ROS) of the fire front as a function of the distance of the fire front from a square array of  $N$  by  $N$  houses each separated by a length  $L$  and burning with a HRR  $Q_0$ . The base case is labelled  $L = 60$  and has  $L = 60$  m,  $N = 5$ , and  $Q_0 = 50$  MW.

### 5.3 Fire-front Propagation

For the spread of a wildfire, it is usual to consider a fire front of arbitrary shape advancing into unburned fuel. Behind the front, the fuel is assumed to be burned, and the front is taken to be thin relative to other dimensions of the problem. The model for the front propagation can then be formulated mathematically in two related but different descriptions. One is the so-called Lagrangian description and the other is an Eulerian description, [44], [47]. In the former formulation, the advance of each Lagrangian particle on the front is related to the empirically determined rate of spread (ROS) of a fire at the locally determined wind speed. It is the most straightforward description and requires following only a one-dimensional, time-dependent array of these Lagrangian particles. The latter formulates the problem as a two-dimensional, time-dependent, convection-diffusion partial differential equation, for which the fire front at any time is a 2D curve representing a constant value of a dependent variable of the problem. According to Sethian [44], this formulation offers some advantages for following the front progression. However, a distinct disadvantage of this formulation is that it requires solution of a PDE in two spatial dimensions and time.

In the description of the spread of wildfires, it is often assumed that the fire-front propagation takes place according to Huygen's principle [48], [49], [21], [47]. Huygen's principle states that the location of a propagating front after a small increment of time can be found from its earlier location by calculating the envelope of the wavelets emitted from all points along the earlier front. Huygen's principle has been widely used in a variety of applications; it was originally developed to explain and calculate optical wave front growth, but has been applied to a other areas as well.

Gwynfor Richards [48], [49] used an approximate form of Huygen's principle to determine the growth of a wildfire front. It was implemented into the operational wildfire prediction code, FARSITE, by Mark Finney [21]. This conceptual model has also been used to describe the spread of fronts in combustion as well as in other applications; see for example, Sethian [44], Kerstein et al [45] and Aldredge [46].

For our purposes, we consider the Lagrangian description, and take as the governing equations, the two

ordinary differential equations (ODEs) describing the propagation of each Lagrangian element in the  $x,y$ -plane.

$$\begin{aligned}\frac{dx}{dt} &= U_x \cdot n_x \\ \frac{dy}{dt} &= U_y \cdot n_y\end{aligned}\tag{4}$$

where  $x,y$  is the location of the Lagrangian element on the fire front,  $U_x, U_y$  are the  $x$  and  $y$  components of the spread-rate vector of the fire front at the location  $(x,y)$ , and  $n_x, n_y$  are the components of the unit normal to the fire front directed toward the unburnt fuel. If the fire front curve at any specified time  $t$  is described by the vector function  $(x(s,t), y(s,t))$ , where  $s$  is a parameter determining the curve.

At each point, the fire front is advanced in the direction normal to the front at a speed determined by the local ROS for the fire. This ROS, in turn, depends on the wind speed at that location. One might regard this procedure as a method-of-characteristics calculation. For computational purposes, the fire front is discretized and then moved incrementally to its new location as described below. We start with an approximation to the normal ROS, and then numerically solve the governing equations. We use the Method of Lines (MOL) and a centered difference scheme for the spatial discretization of the fire line. The discussion below describes a wind-blown grass fire using Eq.(1) to relate the ROS of the fire to the local wind speed.

For a fire front exposed to the velocity field generated by a burning structure of HRR  $Q_0$ , the characteristic length and velocity scales are  $D^*$  and  $V^*$  as discussed above. Let  $r'$  denote the vector distance from the center of the structure to the element of the fire front. The velocity at this point will be  $v(r') = V^*v(r'/D^*)$ , where  $v(r'/D^*)$  is the dimensionless velocity and  $D^*$  is the length scale defined above, and the dimensionless vector distance,  $\vec{r}$ , is  $\vec{r} = \vec{r}'/D^*$ . In addition, we assume that there is a uniform (in space and time) ambient wind  $\vec{V}_a$ , which is added vectorially to the entrainment velocity.

Let the fire line initially be a straight line along the  $x$ -axis, running between  $-L$  and  $L$ . We divide this interval into a discrete set of nodes and use the method of lines (MOL) to solve the equations for the motion of these nodes. Details of the mathematical equations for following the fire front are presented in the Appendix, and solutions are given and discussed in the next section below.

We have developed and solved the simplest example that illustrates the effects of burning structures on fire front propagation. We consider an initially straight fire front parallel to the  $x$ -axis and propagating in the  $y$ -direction. See the schematic diagram in Figure 7.

## 6 Results and Discussion

This simple wind-blown model for WUI fire front propagation depends on several parameters. At this time, only a fraction of these effects have been examined, and then only for a limited set of values for the parameters. In the results presented here, the general formulation for the fire-front propagation over a single period has been specialized even further by taking the  $x$  positions of each node to be fixed. While this assumption might at first seem rather drastic, it has been found that this assumption is reasonably good for a variety of approximations for the ROS vector, and this assumption has been found to make the computations much faster and more robust.

As discussed above, the scaling relations presented earlier are very important. They demonstrate the effects, for example, of housing density, since the spatial period  $2L$  is inversely proportional to the square

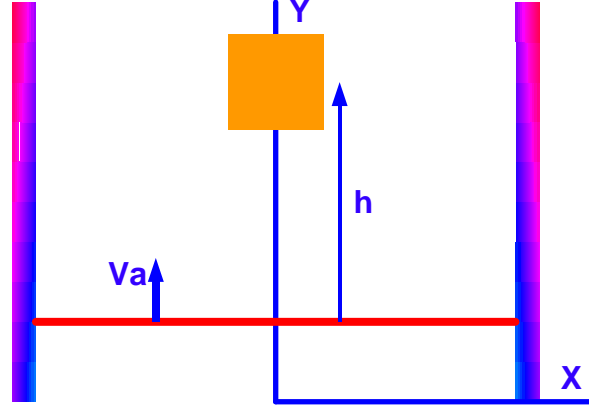


Figure 7: The initial fire line (red), the structure, axes and the periodic boundary conditions at  $x = L$  and  $x = -L$ .

root of this housing density:

$$2L = \sqrt{\frac{A}{n}} \quad (5)$$

where  $n$  is the number of houses and  $A$  is the area considered. It is easy to think of  $A$  being an acre ( $=4047 \text{ m}^2$ ) and  $n$  as the number of houses on that acre, since, in a typical suburban area, a house lot is about  $1/4$  acre. Therefore,  $n = 4$ , and  $2L = 32\text{m}$ . The top curve of the three curves labelled by  $L$  in the right plot in Figure 6 is for  $30 \text{ m}$  (or approximately  $1/4$  the housing density in this typical neighborhood). The second of the three plots ( $L=60 \text{ m}$ ) is for a housing density of 1 house per 4 acres while the third plot ( $L=120 \text{ m}$ ) is for a housing density of 1 house per 16 acres. Recall that these three plots in Figure 6 are for an array of 25 ( $M = N = 5$ ) houses with a HRR  $Q_0 = 50 \text{ MW}$  per house. The remaining two plots show what happens to the base case  $L = 60 \text{ m}$  when each parameter is doubled,  $M = N = 10$  or  $Q_0 = 100 \text{ MW}$  with the other parameters for the base case remaining the same.

Two sets of results for the detailed computations described above are shown in each of Figures 8 and 9. Figure 8 shows plots for a single house while Figure 9 shows plots for the center house in a line of eleven houses. In the left plot of Figure 8, the fire front progression is shown for a structure burning at  $100 \text{ MW}$  intensity with an ambient wind speed of  $2 \text{ m/s}$  blowing toward the top of the diagram. The structure is  $12 \text{ m}$  on a side, the fire front is shown every  $20 \text{ s}$  starting as a straight line  $20 \text{ m}$  below the burning structure and the spatial period is  $2L = 60\text{m}$ . In the plot on the right, the conditions are the same except that the fire front starts initially  $10 \text{ m}$  behind the burning structure; i.e., it is assumed that the fire front has passed the house before it ignites and becomes fully involved.

The model for a single-house case with periodic boundary conditions on the fire front, assumes there are no neighboring burning structures. The periodic BC's are used for convenience only and do not indicate true periodic behavior with respect to the burning structures. When the characteristic length  $2L$  is large enough, the BC's should have little effect on the solution for the fire-front location. Alternately, for the case of a "line of houses" (see Figure 9), the fire front location at various times is shown for the center house, and includes the entrainment wind generated by the house shown and its five neighboring houses on each side.

An advantage of this model is that it is very fast and resolves the fire front with a minimum numbers of nodes, even when several burning structures are assumed involved. For other formulations, resolving the fire front when several burning structures are involved, can be much more computationally intensive and

much less robust.

In each case in Figure 9, the fire front initially is a straight line 30 m below the burning structure. In the left plot, there is an ambient wind of 2 m/s while in the right plot there is a 5 m/s ambient wind. In each plot, the time interval between fire front locations is the same; in the left plot it is 30 s and in the right 20 s. For all cases shown in Figures 8 and 9, the fire front is accelerated as it approaches the structure and is retarded after it passes. However, for the line of houses, the entrainment velocity is enhanced by the accumulative effect of the row of burning structures, thereby accelerating the fire front more as it approaches the burning houses and retarding it more after passing the houses.

Note that the primary influence of the entrainment winds from burning houses on the overall propagation of a grass-fire front is determined by the scaling relations given in Eq.(3) and illustrated in Figure 6. These relations show that the fuel system for WUI fires must include structures as well as vegetation, and that the progression of a grass-fire front (and probably other vegetation fire fronts also) can be altered substantially when a large number of structures are burning. The plots in Figures 8 and 9 show examples of detailed fire-front propagation changes due to burning structures and also show that these changes can be tracked in a computationally inexpensive and robust fashion. Even in such a simple model, several parameters determine the details of the fire-front propagation, and only a few of these variations have been presented here.

## 7 Conclusions

There are several ideas upon which the proposed model are based. These are:

1. The time scale associated with burning grass and burning individual trees is measured in tens of seconds in contrast with the time scale for burning structures, which is measured in hours. Therefore, the coupling between the natural fuel burning and the structure fuel burning is loose. For example, burning houses are shown to influence grass fire propagation, but, excluding ignition (about which nothing has been said here), it is not clear how grass fires influence the burning of houses. Furthermore, because of the disparity in time scales, it will be difficult to utilize a field model such as FDS to compute multiple burning houses and vegetative (WUI) fires over large areas in any detail due to constraints on computational resources. Therefore, using field models for operational guidance on large-area WUI fires in the near term is unlikely.
2. The HRR,  $Q_0$ , of a structure fire determines the strength of its plume and defines a characteristic length scale  $D^*$  and a characteristic velocity scale  $V^*$  for the entrainment of the plume; see the plume model of Baum and McCaffrey [31]. This model requires that the plume stand upright, and, therefore, that the ambient wind  $U_a$ , be less than the characteristic plume velocity ( $U_a < V^*$ ).
3. The ground level entrainment velocity resulting from the plume model of Baum and McCaffrey decays with radial distance  $r$  from the plume as  $r^{-1/3}$  at large distances. This slow decay implies that the entrainment has significant influence over large distances, and the mass-fire study of Baum and McCaffrey recognizes this fact. It shows up here in the large enhancement factor caused by a single burning structure and also in the even larger enhancement factor arising from multiple burning structures! Furthermore, the magnitude of the enhancement factor for multiple burning structures is found to increase significantly as the housing density increases, or as the separation distance between (burning) houses decreases, and also as the HRR for each structure increases!
4. The influence of the entrainment winds from burning houses on the overall propagation of a grass-fire front is demonstrated by the scaling relations given in Eq. (3) and illustrated in Figure 6. These relations

show that the fuel system for WUI fires must include structures as well as vegetation, and that the progression of a grass-fire front (and probably other vegetation fire fronts also) can be altered substantially when a large number of structures are burning. The plots in Figures 8 and 9 show examples of detailed fire-front propagation changes due to burning structures and also show that these changes can be tracked in a computationally inexpensive and robust fashion. Even in such a simple model, several parameters determine the details of the fire-front propagation, and only a few of these variations have been presented here.

5. The selection of a periodic model for a grass-fire front propagating past burning structures provides a simple, convenient example of a WUI fire that allows direct comparison between the line-fire effects and the structure-fire effects; see Tables 5 and 6 for some direct comparisons. This comparison is much more natural than earlier models attempted. The strength of a front is measured by its line-fire intensity  $I$ , its ROS  $U$ , and the energy density of wildland fuel  $hm_f$ . From these quantities and the spatial length  $L$ , all of the necessary parameters can be obtained for comparison with the corresponding quantities obtained from a periodically placed set of structures.

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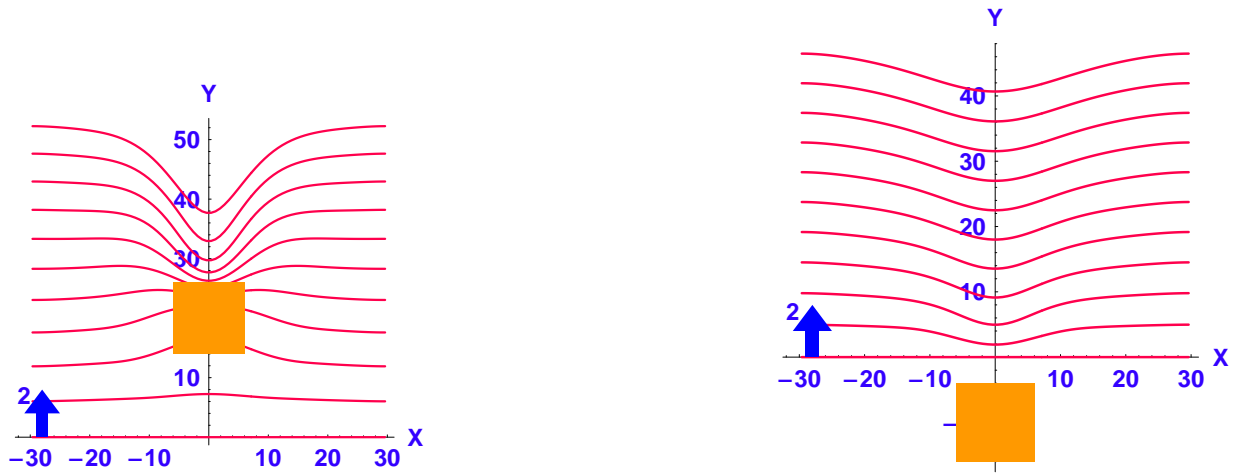


Figure 8: The fire front at several equal time intervals for a burning structure with an ambient wind of 2 m/s, a 100 MW fire and  $L=30$ . LEFT: House is burning before arrival of fire front. RIGHT: House ignites after passage of fire front.

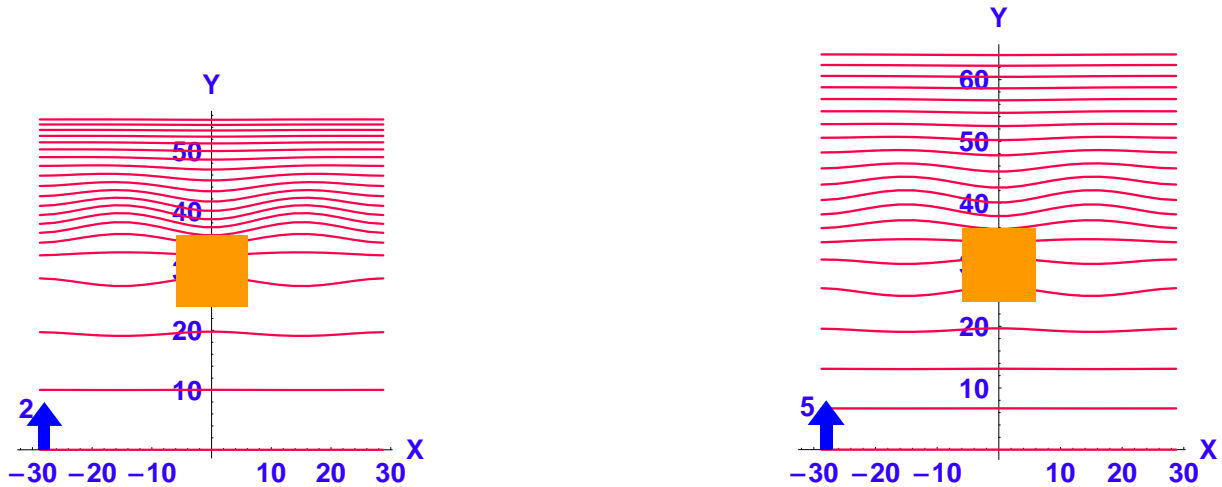


Figure 9: The fire front at several equal time intervals near the center structure of a line of eleven burning structures. LEFT: Ambient wind of 2 m/s. RIGHT: Ambient velocity of 5 m/s.

## Appendix

In this Appendix, Lagrangian equations are presented for determining the fire front propagation in the presence of burning structures. We take as the governing equations, the two ordinary differential equations (ODEs) describing the propagation of each Lagrangian element in the  $x,y$ -plane:

$$\begin{aligned}\frac{dx}{dt} &= U_x \cdot n_x \\ \frac{dy}{dt} &= U_y \cdot n_y\end{aligned}\tag{6}$$

where  $x,y$  is the location of the Lagrangian element on the fire front,  $U_x, U_y$  are the  $x$  and  $y$  components of the spread-rate vector of the fire front at the location  $(x,y)$ , and  $n_x, n_y$  are the components of the unit normal to the fire front directed toward the unburnt fuel. If the fire front curve at any specified time  $t$  is described by the vector function  $(x(s,t), y(s,t))$ , where  $s$  is a parameter determining the curve, then, the unit normal is

$$\begin{aligned}n_x &= \frac{-dy/ds}{\sqrt{(dx/ds)^2 + (dy/ds)^2}} \\ n_y &= \frac{dx/ds}{\sqrt{(dx/ds)^2 + (dy/ds)^2}}\end{aligned}\tag{7}$$

At each point, the fire front is advanced in the direction normal to the front at a speed determined by the local ROS for the fire. This ROS, in turn, depends on the wind speed at that location. One might regard this procedure as a method-of-characteristics calculation. For computational purposes, the fire front is discretized and then moved incrementally to its new location as described below. We start with an approximation to the normal ROS, and then numerically solve the governing equations. We use the Method of Lines (MOL) and a centered difference scheme for the spatial discretization of the fire line. The discussion below describes a wind-blown grass fire using Eq.(1) to relate the ROS of the fire to the local wind speed.

For a fire front exposed to the velocity field generated by a burning structure of HRR  $Q_0$ , the characteristic length and velocity scales are  $D^*$  and  $V^*$  as given in Eq. (2). Let  $\vec{r}'$  denote the vector distance from the center of the structure to the element of the fire front. The velocity at this point will be  $v(r') = V^*v(r'/D^*)$ , where  $v(r'/D^*)$  is the dimensionless velocity, and  $\vec{r} = \vec{r}'/D^*$  is the dimensionless vector distance. We assume that there is a uniform (in space and time) ambient wind  $\vec{V}_a$ , which is added vectorially to the entrainment velocity.

Let the fire line initially be a straight line along the  $x$ -axis, running between  $-L$  and  $L$ . We divide this interval into  $2I$  panels each of length  $\delta$ , where  $\delta = L/I$ . The nodes for the numerical solution are therefore placed at  $x_i = i\delta$ , where the index  $i$  varies between  $-I$  and  $I$ . We use a centered difference discretization to approximate the unit normal at each node along the fire line:

$$\begin{aligned}n_{x,i} &= \frac{-(y_{i+1} - y_{i-1})}{\sqrt{(x_{i+1} - x_{i-1})^2 + (y_{i+1} - y_{i-1})^2}} \\ n_{y,i} &= \frac{(x_{i+1} - x_{i-1})}{\sqrt{(x_{i+1} - x_{i-1})^2 + (y_{i+1} - y_{i-1})^2}}\end{aligned}\tag{8}$$

We will assume at first that the fire front is periodic so that  $x_{-I}(t) = x_I(t)$ , and  $y_{-I}(t) = y_I(t)$ . The assumption of a periodic fire front is convenient.

After discretization, we can write the ordinary differential equations (ODEs) in the interior of the fire front, for  $i = -I + 2, -I + 3, \dots, I - 2, I - 1$ , as

$$\begin{aligned}\frac{dx_i}{dt} &= U_{x,i} \cdot \frac{-(y_{i+1} - y_{i-1})}{\sqrt{(x_{i+1} - x_{i-1})^2 + (y_{i+1} - y_{i-1})^2}} \\ \frac{dy_i}{dt} &= U_{y,i} \cdot \frac{(x_{i+1} - x_{i-1})}{\sqrt{(x_{i+1} - x_{i-1})^2 + (y_{i+1} - y_{i-1})^2}}\end{aligned}\quad (9)$$

Separately we must write the equations for the nodes at the ends of the single period of the fire front, taking into account the assumed periodicity. At the left,

$$\begin{aligned}\frac{dx_{-I+1}}{dt} &= U_{x,-I+1} \cdot \frac{-(y_{-I+2} - y_{-I})}{\sqrt{(x_{-I+2} - x_{-I})^2 + (y_{-I+2} - y_{-I})^2}} \\ \frac{dy_{-I+1}}{dt} &= U_{y,-I+1} \cdot \frac{(x_{-I+2} - x_{-I})}{\sqrt{(x_{-I+2} - x_{-I})^2 + (y_{-I+2} - y_{-I})^2}}\end{aligned}\quad (10)$$

Similarly, at the right

$$\begin{aligned}\frac{dx_I}{dt} &= U_{x,I} \cdot \frac{-(y_{-I+1} - y_{I-1})}{\sqrt{(x_{-I+1} - x_{I-1})^2 + (y_{-I+1} - y_{I-1})^2}} \\ \frac{dy_I}{dt} &= U_{y,I} \cdot \frac{(x_{-I+1} - x_{I-1})}{\sqrt{(x_{-I+1} - x_{I-1})^2 + (y_{-I+1} - y_{I-1})^2}}\end{aligned}\quad (11)$$

As stated above, the initial conditions for these equations are  $y_i(0) = 0$  and  $x_i(0) = \delta * i$  for  $-I \leq i \leq I$ .

We must now approximate  $U_{x,i}, U_{y,i}$ , the wind-driven fire-front ROS vector components at each node of the fire front. For a single burning structure at  $(x = 0, y = H)$ , the induced entrainment velocity components at the node  $i$  are

$$\begin{aligned}V_{x,i} &= \frac{x_i}{\sqrt{x_i^2 + (y_i - H)^2}} V^* v(\sqrt{x_i^2 + (y_i - H)^2} / D^*) \\ V_{y,i} &= \frac{y_i - H}{\sqrt{x_i^2 + (y_i - H)^2}} V^* v(\sqrt{x_i^2 + (y_i - H)^2} / D^*)\end{aligned}\quad (12)$$

as shown above. Let the ambient wind have components  $(V_a \sin \theta_a, V_a \cos \theta_a)$ , where  $V_a$  is the ambient wind speed and  $\theta_a$  is the angle of the ambient wind relative to north. Then the total wind speed at node  $i$  is

$$\begin{aligned}V_{t,x,i} &= V_a \sin \theta_a + \frac{x_i}{\sqrt{x_i^2 + (y_i - H)^2}} V^* v(\sqrt{x_i^2 + (y_i - H)^2} / D^*) \\ V_{t,y,i} &= V_a \cos \theta_a + \frac{y_i - H}{\sqrt{x_i^2 + (y_i - H)^2}} V^* v(\sqrt{x_i^2 + (y_i - H)^2} / D^*)\end{aligned}\quad (13)$$

Defining the total wind speed as  $V_{t,i} = \sqrt{V_{t,x,i}^2 + V_{t,y,i}^2}$ , and using the relation defined above between the wind speed at any location and the wind-driven fire-front speed for Australian grass, we find

$$U_i \equiv r_w = r_0(1 + c_f \cdot V_{t,i}) \quad (14)$$

where  $r_0 = 0.165$  m/s is the ROS with no wind and  $c_f = 0.324$  is the coefficient, both introduced earlier in Eq.(1).